Compound Angles - Calculations and Jig for Making for N-sided Pyramids

by

Tony Beugelsdijk

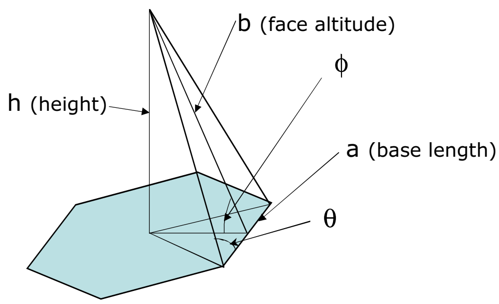

Figure 1. Definition of angles and sides.

At my recent AWA presentation on scroll saw clockmaking, many members asked about the equations I developed for calculating the compound angle settings for construction of an n-sided pyramid. For some of the more complex clocks, you'll inevitably be called upon to make 4-, 6-, and 8-sided pyramids of varying height. This article summarizes the equations and, through several examples, shows how to set up a table saw to make the cuts. I also show the jig I made to hold the work pieces safely and accurately.

The equations are generic in that they apply for any number of sides (greater or equal to 3) and any base length and height. They can be used for example, to make n n-sided staved container, with or without handles.

To define the labeling I used, refer to Figure 1 which shows a 6-sided pyramid. As a designer you will need to decide how many sides, n, the pyramid will have, as well as the length of the base of a face triangle, a, and the overall height of the pyramid, h. Once you know from a set of drawings or have set these parameters, the pyramid is geometrically defined and the compound cutting angles can be calculated.

The equations below also calculate the altitude of the face triangle, b, as shown so that a paper pattern for application to the face can be made.

To start, it is helpful to define a constant, κ, as

$$\kappa = \tan\left(\tfrac{\pi}{2} - \tfrac{\pi}{n}\right) \qquad (1)$$

Note that κ is only dependent upon the number of sides of the pyramid. For example, if n=4, $\kappa=\tan(\pi/4)$ or 1.000 and for n=6, $\kappa=\tan(\pi/6)$ or 1.732.

I should mention that most scientific calculators are set to accept angles expressed in radians. There are pi (π) or 3.1416 radians in 180 degrees. Thus to convert an angle from degrees to radians, multiply by (π/180) and, vice versa, to convert an angle from radians to degrees multiply by (180/π). Most trig tables use angles so this conversion is not needed.

The first calculation is the angle the face triangle makes from the horizontal plane of the pyramid base. I have labeled this angle, ϕ, in Figure 1 and it is given by:

$$\phi = \tan^{-1}\left(\tfrac{2h}{a\kappa}\right) \qquad (2)$$

As an example, if you choose a = 2 and h=6 inches, for a 6-sided pyramid, $\phi=\tan^{-1}(3.46)$ or 73.90 degrees. This angle is the first that you will cut on the table saw and, for most saws, you will need to set your blade angle to the complement of

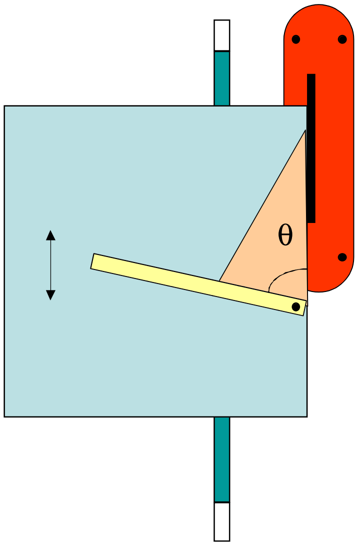

Figure 2. Compound angle jig.

ϕ, so for the first example with $\phi = 80.54$ degrees, you will need to set the blade to 90.00-80.54 or 9.46 degrees. The fence is set a distance greater than b from the blade. When the sides are cut later, the excess height is cut off.

A word here about accuracy: the equations deliver as many decimal points as needed, however, your ability to set the saw blade to the angle calculated is probably only good to within ±0.25 degrees on most saws. This is accurate enough for most applications.

The next 2 angles are the base angle of the face isosceles triangle, θ, and the compound angle of the side of this same triangle, τ.

These angles are given by:

$$\theta = \tan^{-1}\sqrt{(\tfrac{2h}{a})^2 + \kappa^2} \qquad (3)$$

and

$$\tau = \tan^{-1}(\tfrac{1}{\tan(\phi)\cos(\theta)}) \qquad (4)$$

Returning to our previous example where n=6, a=2 and h=6,

$$\theta = \tan^{-1}\sqrt{(\tfrac{2\bullet 6}{2})^2 + 1.732^2}$$

$\theta = \tan^{-1}(6.24)$ or 80.90 degrees.

$$\tau = \tan^{-1}(\tfrac{1}{\tan(73.90)\cos(80.90)}),$$

$\tau = \tan^{-1}(1.07)$ or 61.29 degrees

Each of the n face triangles can now be cut using these angles. I have found it best to construct the simple jig shown in Figure 2. An end view of the same jig is shown in Figure 3. Also, the angle, τ, can be opened by a few tenths of a degree to allow for glue squeeze out on the interior of the pyramid and to insure a crisp outside edge. In the example above, setting the saw angle to 61.5 degrees will yield good results.

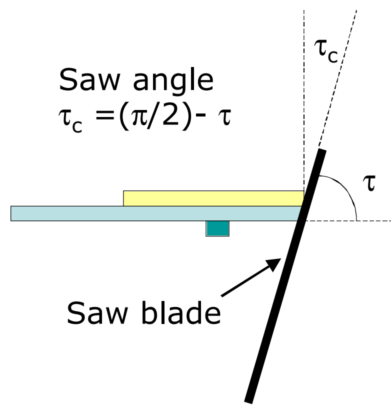

Figure 3. Compound angle jig (end view).

A piece of wood can now be selected. Orient the piece so that the grain direction will point to the pyramid's peak. The cutting sequence for a face triangle is as follows: 1) cut a piece of wood with the angle ϕ along the bottom edge, (this edge will become part of the base of the pyramid), 2) place the edge just cut onto the horizontal bar of the jig shown in Figure 2 (with the horizontal bar adjusted to the angle θ, as shown), 3) set the saw blade angle to the complement of τ as shown in Figure 3 (i.e., 90-τ). Make the cut along one edge of the face triangle. With the first edge cut, re-adjust the horizontal bar of the jig to -θ, and cut the opposite edge completing the face triangle.

It is helpful to mount hold down clamps to the jig for safety and to insure that the part doesn't move during a cut.

The final equation gives the altitude, b, of the face isosceles triangle as shown in Figure 1. This is useful for drawing a paper pattern.

$$b = \sqrt{h^2 + \left(\frac{a\kappa}{2}\right)^2} \qquad (5)$$

Following through with our example:

$$b = \sqrt{6^2 + \left(\frac{2 \bullet 1.732}{2}\right)^2}$$

b = 6.24

Thus each of the face triangles has a base of 2 inches and an altitude of 6.24 inches.

The above equations are available in an Excel spreadsheet. A copy of the spreadsheet for various number of sides is shown in Table 1 below. The example followed in this article is summarized in the third row of this Table.

Pyramid Angle Spreadsheet

n	a	h	κ	θ_{min}	θ	θ_{max}	ϕ	τ	b
3	6	6	0.577	30.00	64.34	90.00	73.90	33.69	6.24
4	2	6	1.000	45.00	80.66	90.00	80.54	45.77	6.08
6	2	6	1.732	60.00	80.90	90.00	73.90	61.29	6.24
8	2	6	2.414	67.50	81.21	90.00	68.08	69.21	6.47
12	2	6	3.732	75.00	81.94	90.00	58.12	77.30	7.07

Table 1.

Often it is desirable to have the base of the pyramid fit within a circle of fixed radius. Figure 4 shows a 6-sided pyramid base circumscribed within a circle of radius r.

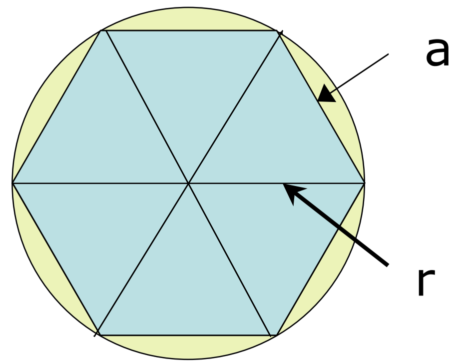

Figure 4. Circle circumscribed around a pyramidal base.

Before the equations above can be used, it is necessary to calculate the length of the base of the face triangle, a.

Given a known radius, r, the side length, a, can be calculated from:

$$a = 2r\sin(\pi/n) \qquad (7)$$

For n=6 sides, for example, a=r. Therefore a circle with a 2 in radius has a face triangle base of 2 in. (In the case of n=6 only a=r).

If you have any further questions or if I can help in any way, please feel free to contact me at: beugelsdijk@lanl.gov.